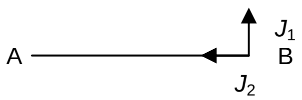
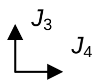
1(i)	$m = \frac{4}{3}\pi r^3 \rho$	M1	Expression for *m*	
	$\frac{\mathrm{d}m}{\mathrm{d}t} = 4\pi r^2 \rho \frac{\mathrm{d}r}{\mathrm{d}t}$	M1	Relate $\frac{\mathrm{d}m}{\mathrm{d}t}$ to $\frac{\mathrm{d}r}{\mathrm{d}t}$	
	$\lambda \cdot 4\pi r^2 = 4\pi r^2 \rho \frac{\mathrm{d}r}{\mathrm{d}t}$	M1	Use of $\frac{\mathrm{d}m}{\mathrm{d}t}$ proportional to surface area	
	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\lambda}{\rho} = k$	E1	Accept alternative symbol for constant if used correctly (here and subsequently)	
	$r = r_0 + kt$	M1	Integrate and use condition	
	$m = \frac{4}{3}\pi\rho(r_0 + kt)^3$	A1		
				6
(ii)	$\frac{\mathrm{d}}{\mathrm{d}t}(mv) = mg$	M1	N2L	
	$mv = \int mg \ \mathrm{d}t = \int \frac{4}{3}\pi\rho(r_0 + kt)^3 g \ \mathrm{d}t$	M1	Express *mv* as an integral	
	$= \frac{4}{3}\pi\rho g\left[\frac{1}{4k}(r_0 + kt)^4 + c\right]$	M1	Integrate	
	$t = 0, v = 0 \Rightarrow c = -\frac{1}{4k} r_0^{\ 4}$	M1	Use condition	
	$\frac{4}{3}\pi\rho(r_0 + kt)^3 v = \frac{4}{3}\pi\rho g \cdot \frac{1}{4k}\left[(r_0 + kt)^4 - r_0^{\ 4}\right]$	M1	Substitute for *m*	
	$v = \frac{g}{4k}\left[r_0 + kt - \frac{r_0^{\ 4}}{(r_0 + kt)^3}\right]$	A1		
				6
2(i)	$AP = 2a\cos\theta$	M1	Attempt AP in terms of θ	
	$PB = \frac{5}{2}a - 2a\cos\theta$	E1		
	$V = -mg \cdot PB - mg \cdot PA\cos\theta$	M1	Attempt *V* in terms of θ	
	$= -mg\left(\frac{5}{2}a - 2a\cos\theta\right) - mg\left(2a\cos\theta\right)\cos\theta$			
	$= -mga\left(2\cos^2\theta - 2\cos\theta + \frac{5}{2}\right)$	E1		
				4
(ii)	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = mga\sin\theta\left(4\cos\theta - 2\right)$	M1	Differentiate	
	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = 0 \Rightarrow \sin\theta = 0$ or $\cos\theta = \frac{1}{2}$	M1	Solve	
	$\Rightarrow \theta = 0$ or $\pm\frac{1}{3}\pi$	A1	For 0 and either of $\frac{1}{3}\pi$ or $-\frac{1}{3}\pi$	
	$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = mga\sin\theta\left(-4\sin\theta\right) + mga\cos\theta\left(4\cos\theta - 2\right)$	M1	Differentiate again	
		A1		
	$\theta = 0 \Rightarrow \frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = 2mga > 0 \Rightarrow$ stable	M1	Consider sign of V'' in one case	
		F1	Correct deduction for one value of θ	
	$\theta = \pm\frac{1}{3}\pi \Rightarrow \frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -3mga < 0 \Rightarrow$ unstable	F1	Correct deduction for another value of θ	
			N.B. Each F mark is dependent on both M marks. To get both F marks, the two values of θ must be physically possible (i.e. in the first or fourth quadrant) and not be equivalent or symmetrical positions.	
				8

3(i)	$P = Fv = mv\frac{dv}{dx}v$	M1	Use of $P = Fv$	
	$v^2\frac{dv}{dx} = 0.0004\left(10\,000v + v^3\right)$	A1	Or equivalent	
	$\int \frac{v}{10\,000 + v^2}\,dv = \int 0.0004\,dx$	M1	Separate variables	
	$\frac{1}{2}\ln\left\|10\,000 + v^2\right\| = 0.0004x + c$	M1	Integrate	
	$v^2 = Ae^{0.0008x} - 10\,000$	M1	Rearrange	
	$x = 0, v = 0 \Rightarrow A = 10\,000$	M1	Use condition	
	$v = 100\sqrt{e^{0.0008x} - 1}$	A1		
	$x = 900 \Rightarrow v = 102.7 > 80$ so successful			
	or $v = 80 \Rightarrow x = 618.37 < 900$ so successful	E1	Show that their *v* implies successful take off	
				8
(ii)	$v\frac{dv}{dt} = 0.0004\left(10\,000v + v^3\right)$	F1	Follow previous DE	
	$\int \frac{1}{10\,000 + v^2}\,dv = \int 0.0004\,dt$	M1	Separate variables	
	$\frac{1}{100}\tan^{-1}\left(\frac{1}{100}v\right) = 0.0004t + k$	M1	Integrate	
		A1		
	$t = 0, v = 0 \Rightarrow k = 0$	M1	Use condition	
	$\Rightarrow v = 100\tan(0.04t)$	E1	Clearly shown	
	$v \to \infty$ at finite time suggests model invalid	B1		
				7
(iii)	$t = 11 \Rightarrow v = 47.0781$	B1	At least 3sf	
	Hence maximum $P = 230.049m$	M1	Attempt to calculate maximum *P*	
	$v = 47.0781 \Rightarrow x = 250.237$	M1	Use solution in (i) to calculate *x*	
	$v^2\frac{dv}{dx} = 230.049$	M1	Set up DE for $t \geqslant 11$. Constant acceleration formulae $\Rightarrow$ M0.	
	$\frac{1}{3}v^3 = 230.049x + B$	M1	Separate variables and integrate	
		F1	Follow their maximum *P* (condone no constant)	
	$v = 47.0781, x = 250.237 \Rightarrow B = -22786.3$	M1	Use condition on *x*, *v* (not $v = 0$, not $x = 0$ unless clearly compensated for when making conclusion). Constant acceleration formulae $\Rightarrow$ M0.	
	$v = 80 \Rightarrow x = 840.922$ or $x = 900 \Rightarrow v = 82.0696$	M1	Relevant calculation. Must follow solving a DE.	
	so successful	A1	All correct (accept 2sf or more)	
				9

4(i)	Considering elements of length $\delta x \Rightarrow I = \int_0^{2a} \rho x^2 \, dx$	M1	Set up integral	
	$= \frac{M}{8a^2} \int_0^{2a} \left(5ax^2 - x^3\right) dx$	M1	Substitute for ρ in predominantly correct integral	
	$= \frac{M}{8a^2} \left[\frac{5}{3} ax^3 - \frac{1}{4} x^4 \right]_0^{2a}$	M1	Integrate	
	$= \frac{7}{6} Ma^2$	E1		
	Considering elements of length $\delta x \Rightarrow M\bar{x} = \int_0^{2a} \rho x \, dx$	M1	Set up integral	
	$= \frac{M}{8a^2} \int_0^{2a} \left(5ax - x^2\right) dx$	M1	Substitute for ρ in predominantly correct integral	
	$= \frac{M}{8a^2} \left[\frac{5}{2} ax^2 - \frac{1}{3} x^3 \right]_0^{2a}$	M1	Integrate	
	$\bar{x} = \frac{11}{12} a$	E1		
				8
(ii)	$\frac{1}{2} I \dot{\theta}^2 = Mg \cdot \frac{11}{12} a (1 - \cos\theta)$	M1	KE term in terms of angular velocity	
		B1	$\pm Mg \cdot \frac{11}{12} a \cos\theta$ seen	
		M1	energy equation	
	$\dot{\theta} = \sqrt{\frac{11g}{7a}(1 - \cos\theta)}$	A1		
				4
(iii)	 $\theta = \frac{1}{2}\pi \Rightarrow \dot{\theta} = \sqrt{\frac{11g}{7a}}$	F1	Their $\dot{\theta}$ at $\theta = \frac{1}{2}\pi$	
	$2a \cdot (-J_1) = I\left(0 - \sqrt{\frac{11g}{7a}}\right)$	M1	Use of angular momentum	
		A1	Correct equation (their $\dot{\theta}$)	
	$J_1 = \frac{1}{12} M \sqrt{77ag}$	E1		
	$J_2 = \frac{1}{12} M \sqrt{77ag}$	B1	Correct answer or follow their J_1	
				5
(iv)	 $J_4 = J_2$	M1	Consider horizontal impulses	
	$= \frac{1}{12} M \sqrt{77ag}$	F1	Follow their J_2	
	$J_3 + J_1 = M \cdot \frac{11}{12} a \sqrt{\frac{11g}{7a}}$	M1	Vertical impulse-momentum equation	
		M1	Use of $r\dot{\theta}$	
	$J_3 = \frac{1}{21} M \sqrt{77ag}$	A1	cao	
	angle $= \tan^{-1}\left(\frac{J_3}{J_4}\right) = \tan^{-1}\left(\frac{\frac{1}{21} M \sqrt{77ag}}{\frac{1}{12} M \sqrt{77ag}}\right)$	M1	Must substitute	
	$= \tan^{-1}\left(\frac{4}{7}\right) \approx 0.519 \text{ rad} \approx 29.7°$	A1	cao (any correct form)	
				7

1(i)	$x = \text{PB}$	M1	May be implied	
	$x = \sqrt{a^2 + y^2}$	A1		
	$V = \frac{1}{2}kx^2 - mgy$	M1	EPE term	
		M1	GPE term	
	$= \frac{1}{2}k\left(a^2 + y^2\right) - mgy$	A1	cao	
				5
(ii)	$\frac{\mathrm{d}V}{\mathrm{d}y} = ky - mg$	M1	Differentiate their V	
	equilibrium $\Rightarrow \frac{\mathrm{d}V}{\mathrm{d}y} = 0$	B1	Seen or implied	
	$\Rightarrow y = \frac{mg}{k}$	A1	cao	
	$\frac{\mathrm{d}^2V}{\mathrm{d}y^2} = k > 0$	M1	Consider sign of V'' (or V' either side)	
	$\Rightarrow$ stable	E1	Complete argument	
				5
(iii)	$R = T\sin P\hat{B}A = k \cdot PB \cdot \frac{a}{PB}$	M1	Use Hooke's law and resolve	
	$= ka$	A1		
				2
2(i)	$\frac{\mathrm{d}}{\mathrm{d}t}(mv) = 0 \Rightarrow mv$ constant	M1	Or no external forces $\Rightarrow$ momentum conserved, or attempt using δ terms.	
	hence $mv = m_0u$	A1		
	$\frac{\mathrm{d}m}{\mathrm{d}t} = k$	B1	$\frac{\mathrm{d}m}{\mathrm{d}t} = k$ seen	
	$\Rightarrow m = m_0 + kt$	B1	$m_0 + kt$ stated or clearly used as mass	
	$v = \frac{m_0u}{m} = \frac{m_0u}{m_0 + kt}$	E1	Complete argument (dependent on all previous marks and $m_0 + kt$ derived, not just stated)	
	$x = \int \frac{m_0u}{m_0 + kt}\,\mathrm{d}t$	M1	Integrate v	
	$= \frac{m_0u}{k}\ln\left(m_0 + kt\right) + A$	A1	cao	
	$x = 0, t = 0 \Rightarrow A = -\frac{m_0u}{k}\ln m_0$	M1	Use condition	
	$x = \frac{m_0u}{k}\ln\left(\frac{m_0 + kt}{m_0}\right)$	A1	cao	
				9
(ii)	$v = \frac{1}{2}u \Rightarrow m_0 + kt = 2m_0$	M1	Attempt to calculate value of m or t	
	$\Rightarrow x = \frac{m_0u}{k}\ln\left(\frac{2m_0}{m_0}\right)$	M1	Substitute their m or t into x	
	$\Rightarrow x = \frac{m_0u}{k}\ln 2$	F1	$t = \frac{m_0}{k}$ or $m = 2m_0$ in their x	
				3

3(i)	$I = \int_{-a}^{a} \rho x^2 \, dx$	M1 A1	Set up integral Or equivalent	
	$\rho = \frac{m}{2a}$	M1	Use mass per unit length in integral or I	
	$I = \frac{m}{2a}\left[\frac{1}{3}x^3\right]_{-a}^{a}$	M1	Integrate	
	$= \frac{1}{6}ma^2 - -\frac{1}{6}ma^2$	M1	Use limits	
	$\frac{1}{3}ma^2$	E1	Complete argument	
				6
(ii)	$I_{\text{rod}} = \frac{1}{3} \times 1.2 \times 0.4^2 + 1.2 \times 0.4^2$	M1 A1	Use $\frac{1}{3}ma^2$ or $\frac{4}{3}ma^2$ Rod term(s) all correct	
	$I_{\text{sphere}} = \frac{2}{5} \times 2 \times 0.1^2 + 2 \times 0.9^2$	M1 M1 A1	Use formula for sphere Use parallel axis theorem Sphere terms all correct	
	$I = I_{\text{rod}} + I_{\text{sphere}} = 1.884$	M1 A1	Add moment of inertia for rod and sphere cao	
				7
(iii)	$\frac{1}{2}I\dot{\theta}^2 - 1.2g \times 0.4\cos\theta - 2g \times 0.9\cos\theta$ $= -1.2g \times 0.4\cos\alpha - 2g \times 0.9\cos\alpha$	M1 M1 M1 A1	Use energy KE term Reasonable attempt at GPE terms All terms correct (but ignore signs)	
	$\dot{\theta}^2 = \frac{4.56g}{1.884}(\cos\theta - \cos\alpha)$	M1 F1	Rearrange Only follow an incorrect I	
				6
(iv)	$2\dot{\theta}\ddot{\theta} = \frac{4.56g}{1.884}(-\sin\theta\,\dot{\theta})$ or $I\ddot{\theta} = -1.2g \times 0.4\sin\theta - 2g \times 0.9\sin\theta$	M1	Differentiate, or use moment $= I\ddot{\theta}$	
		F1	Equation for $\ddot{\theta}$ (only follow their I or $\dot{\theta}^2$)	
	$\sin\theta \approx \theta \Rightarrow \ddot{\theta} = -11.86\theta$	M1	Use small angle approximation (in terms of θ)	
	i.e. SHM	E1	All correct (for their I) and make conclusion	
	$T \approx \frac{2\pi}{\sqrt{11.86}} \approx 1.82$	F1	$\frac{2\pi}{\text{their } \omega}$	
				5

4(i)	$2v\dfrac{dv}{dx} = 2 - 8v^2$	M1 A1	N2L	
	$\int \dfrac{v}{1-4v^2}\,dv = \int dx$	M1	Separate	
	$-\frac{1}{8}\ln\left\lvert 1-4v^2\right\rvert = x + c_1$	A1	LHS	
	$x = 0, v = 0 \Rightarrow c_1 = 0$	M1	Use condition	
	$1 - 4v^2 = e^{-8x}$	M1	Rearrange	
	$v^2 = \frac{1}{4}\left(1 - e^{-8x}\right)$	E1	Complete argument	
				7
(ii)	$F = 2 - 8v^2 = 2 - 2\left(1 - e^{-8x}\right)$	M1	Substitute given v^2 into F	
	$= 2e^{-8x}$	A1	cao	
	Work done = $\int_0^2 F\,dx$	M1	Set up integral of F	
	$= \int_0^2 2e^{-8x}\,dx$	A1	cao	
	$= \left[-\frac{1}{4}e^{-8x}\right]_0^2$	M1	Integrate	
	$= \frac{1}{4}\left(1 - e^{-16}\right)$	A1	Accept $\frac{1}{4}$ or 0.25 from correct working	
				6
(iii)	$2\dfrac{dv}{dt} = 2 - 8v^2$	M1	N2L	
	$\frac{1}{4}\int \dfrac{1}{\frac{1}{4} - v^2}\,dv = \int dt$	M1	Separate	
	$\frac{1}{4}\ln\left\lvert\dfrac{\frac{1}{2}+v}{\frac{1}{2}-v}\right\rvert = t + c_2$	A1	LHS	
	$t = 0, v = 0 \Rightarrow c_2 = 0$	M1	Use condition	
	$\dfrac{\frac{1}{2}+v}{\frac{1}{2}-v} = e^{4t}$	M1	Rearrange (remove log)	
	$1 + 2v = e^{4t}(1 - 2v)$			
	$2v\left(1 + e^{4t}\right) = e^{4t} - 1$	M1	Rearrange (v in terms of t)	
	$v = \frac{1}{2}\left(\dfrac{e^{4t}-1}{e^{4t}+1}\right) = \frac{1}{2}\left(\dfrac{1-e^{-4t}}{1+e^{-4t}}\right)$	E1	Complete argument	
				7
(v)	$t = 1 \Rightarrow v = 0.4820$	B1		
	$t = 2 \Rightarrow v = 0.4997$	B1		
	Impulse $= mv_2 - mv_1$	M1	Use impulse-momentum equation	
	= 0.0353	A1	Accept anything in interval [0.035, 0.036]	
				4

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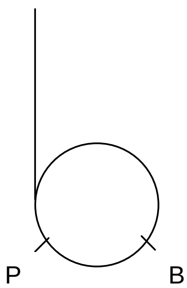
1(i)	If δm is change in mass over time δt PCLM $mv=(m+\delta m)(v+\delta v)+\|\delta m\|(v-u)$ [N.B. $\delta m<0$]	M1	Change in momentum over time δt
	$(m+\delta m)\frac{\delta v}{\delta t}+u\frac{\delta m}{\delta t}=0 \Rightarrow m\frac{\mathrm{d}v}{\mathrm{d}t}=-u\frac{\mathrm{d}m}{\mathrm{d}t}$	M1 A1	Rearrange to produce DE Accept sign error
	$\frac{\mathrm{d}m}{\mathrm{d}t}=-k \Rightarrow m=m_0-kt$	M1	Find m in terms of t
	$\Rightarrow (m_0-kt)\frac{\mathrm{d}v}{\mathrm{d}t}=uk$	E1	Convincingly shown
			5
(ii)	$v=\int \frac{uk}{m_0-kt}\mathrm{d}t$	M1	Separate and integrate
	$=-u\ln(m_0-kt)+c$	A1	cao (allow no constant)
	$t=0, v=0 \Rightarrow c=u\ln m_0$	M1	Use initial condition
	$v=u\ln\left(\frac{m_0}{m_0-kt}\right)$	A1	All correct
			4
(iii)	$m=\frac{1}{3}m_0 \Rightarrow m_0-kt=\frac{1}{3}m_0$	M1 A1	Find expression for mass or time Or $t=2m_0/3k$
	$\Rightarrow v=u\ln 3$	A1	
			3

2(i)	$P=Fv$	M1	Used, not just quoted
	$=mv\frac{\mathrm{d}v}{\mathrm{d}x}v$	M1	Use N2L and expression for acceleration
	$\Rightarrow mv^2\frac{\mathrm{d}v}{\mathrm{d}x}=m\left(k^2-v^2\right)$	A1	Correct DE
	$\Rightarrow \frac{v^2}{k^2-v^2}\frac{\mathrm{d}v}{\mathrm{d}x}=1$	M1	Rearrange
	$\Rightarrow \left(\frac{k^2}{k^2-v^2}-1\right)\frac{\mathrm{d}v}{\mathrm{d}x}=1$	E1	Convincingly shown
	$\int\left(\frac{k^2}{k^2-v^2}-1\right)\mathrm{d}v=\int \mathrm{d}x$	M1	Separate and integrate
	$\frac{1}{2}k\ln\left(\frac{k+v}{k-v}\right)-v=x+c$	A1	LHS
	$x=0, v=0 \Rightarrow c=0$	M1	Use condition
	$x=\frac{1}{2}k\ln\left(\frac{k+v}{k-v}\right)-v$	A1	cao
			9
(ii)	Terminal velocity when acceleration zero $\Rightarrow v=k$	M1 A1	
	$v=0.9k \Rightarrow x=\frac{1}{2}k\ln\left(\frac{1.9}{0.1}\right)-0.9k=\left(\frac{1}{2}\ln 19-0.9\right)k\approx$ 0.572k	F1	Follow their solution to (i)
			3

3(i)	$M = \int_0^a k(a+r)2\pi r\,\mathrm{d}r$	M1 M1	Use circular elements (for *M* or *I*) Integral for mass	
	$= 2k\pi\left[\frac{1}{2}ar^2 + \frac{1}{3}r^3\right]_0^a$	M1 A1	Integrate (for *M* or *I*) For […]	
	$= \frac{5}{3}k\pi a^3$	E1		
	$I = \int_0^a k(a+r)2\pi r \cdot r^2\,\mathrm{d}r$	M1	Integral for *I*	
	$= 2k\pi\left[\frac{1}{4}ar^4 + \frac{1}{5}r^5\right]_0^a$	A1	For […]	
	$= \frac{9}{10}k\pi a^5$	A1	cao	
	$= \frac{27}{50}Ma^2$	E1	Complete argument (including mass)	
				9
(ii)	$I = 13.5$	B1	Seen or used (here or later)	
	$0.625 \times 50 = I\omega$	M1 M1	Use angular momentum Use moment of impulse	
	$\Rightarrow \omega \approx 2.31$	A1	cao	
				4
(iii)	$\ddot{\theta} = \frac{30-2.31}{20} \approx 1.38$	M1	Find angular acceleration	
	Couple $= I\ddot{\theta}$	M1	Use equation of motion	
	≈ 18.7	F1	Follow their ω and *I*	
				3
(iv)	$I\ddot{\theta} = -3\dot{\theta}$	B1	Allow sign error and follow their *I* (but not *M*)	
	$I\frac{\mathrm{d}\dot{\theta}}{\mathrm{d}t} = -3\dot{\theta}$	M1	Set up DE for $\dot{\theta}$ (first order)	
	$\int \frac{\mathrm{d}\dot{\theta}}{\dot{\theta}} = \int -\frac{3}{I}\mathrm{d}t$	M1	Separate and integrate	
	$\ln\lvert\dot{\theta}\rvert = -\frac{t}{4.5} + c$	B1	$\ln(\text{multiple of } \dot{\theta})$ seen	
	$\dot{\theta} = A\mathrm{e}^{-t/4.5}$	M1	Rearrange, dealing properly with constant	
	$t = 0, \dot{\theta} = 30 \Rightarrow A = 30$	M1	Use condition on $\dot{\theta}$	
	$\dot{\theta} = 30\,\mathrm{e}^{-t/4.5}$	A1		
				7
(v)	Model predicts $\dot{\theta}$ never zero in finite time.	B1		
				1

4(i)	$V=\frac{1}{2}\left(\frac{mg}{10a}\right)(a\theta)^2+mga\cos\theta$ (relative to centre of pulley)	M1	EPE term	
		B1	Extension $=a\theta$	
		M1	GPE relative to any zero level	
		A1	(± constant)	
	$\frac{dV}{d\theta}=\frac{1}{2}\left(\frac{mg}{10a}\right)\cdot 2a^2\theta-mga\sin\theta$	M1	Differentiate	
	$\frac{dV}{d\theta}=mga\left(\frac{1}{10}\theta-\sin\theta\right)$	E1		
				6
(ii)	$\theta=0\Rightarrow\frac{dV}{d\theta}=mga\left(\frac{1}{10}(0)-\sin 0\right)=0$	M1	Consider value of $\frac{dV}{d\theta}$	
	hence equilibrium	E1		
	$\frac{d^2V}{d\theta^2}=mga\left(\frac{1}{10}-\cos\theta\right)$	M1	Differentiate again	
		A1		
	$V''(0)=-0.9mga<0$	M1	Consider sign of V''	
	hence unstable	E1	V'' must be correct	
				6
(iii)	If the pulley is smooth, then the tension in the string is constant.	B1		
	Hence the EPE term is valid.	B1		
				2
(iv)	Equilibrium positions at $\theta=2.8$,	B1	One correct	
	$\theta=7.1$	B1	All three correct, no extras	
	and $\theta=8.4$		Accept answers in [2.7,3.0), [7,7.2], [8.3,8.5]	
	From graph, $V''(2.8)=mgaf'(2.8)>0$	M1	Consider sign of V'' or f'	
	hence stable at $\theta=2.8$	A1	Accept no reference to V'' for one conclusion but other two must relate to sign of V'', not just f'.	
	$V''(7.1)=mgaf'(7.1)<0\Rightarrow$ unstable at $\theta=7.1$	A1		
	$V''(8.4)=mgaf'(8.4)>0\Rightarrow$ stable at $\theta=8.4$	A1		
				6
(v)		B1	P in approximately correct place	
		B1	B in approximately correct place	2
(vi)	If $\theta<0$ then expression for EPE not valid	M1		
	hence not necessarily an equilibrium position.	A1		
				2

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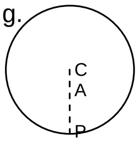
1(i)	$\frac{d}{dt}(mv) = mg$	B1	Seen or implied	
	$\Rightarrow \frac{dm}{dt}v + m\frac{dv}{dt} = mg$	M1	Expand	
	$\Rightarrow \frac{mg}{2(v+1)}v + m\frac{dv}{dt} = mg$	M1	Use $\frac{dm}{dt} = \frac{mg}{2(v+1)}$	
	$\Rightarrow \frac{dv}{dt} = g\left(1 - \frac{v}{2(v+1)}\right) = g\left(\frac{v+2}{2(v+1)}\right)$			
	$\Rightarrow \left(\frac{v+1}{v+2}\right)\frac{dv}{dt} = \frac{1}{2}g$	M1	Separate variables (oe)	
	$\Rightarrow \left(1 - \frac{1}{v+2}\right)\frac{dv}{dt} = \frac{1}{2}g$	E1		5
(ii)	$\int\left(1 - \frac{1}{v+2}\right)dv = \int\frac{1}{2}g\,dt$	M1	Integrate	
	$v - \ln\lvert v+2\rvert = \frac{1}{2}gt + c$	A1	LHS	
	$t = 0, v = 0 \Rightarrow -\ln 2 = c$	M1	Use condition	
	$v - \ln\lvert v+2\rvert = \frac{1}{2}gt - \ln 2$			
	$t = \frac{2}{g}(v - \ln\lvert v+2\rvert + \ln 2)$	A1		
	$v = 10 \Rightarrow t \approx 1.68$	B1		5
(iii)	As t gets large, v gets large	M1		
	So $\frac{dv}{dt} \to \frac{1}{2}g$ (i.e. constant)	A1	Complete argument	2
2(i)	$V = -mg \cdot 2a\sin\theta + \frac{\frac{1}{2}mg}{2a}(4a\sin\theta - a)^2$	B1	GPE	
		M1 A1	Reasonable attempt at EPE EPE correct	
	$\frac{dV}{d\theta} = -2mga\cos\theta + \frac{mg}{2a}(4a\sin\theta - a)\cdot 4a\cos\theta$	M1	Differentiate	
	$= -2mga\cos\theta + 2mga\cos\theta(4\sin\theta - 1)$ $= 4mga\cos\theta(2\sin\theta - 1)$	E1	Complete argument	5
(ii)	$\frac{dV}{d\theta} = 0$	M1	Set derivative to zero	
	$\Leftrightarrow \cos\theta = 0$ or $\sin\theta = \frac{1}{2}$	M1	Solve	
	$\Leftrightarrow \theta = \frac{1}{2}\pi$ or $\frac{1}{6}\pi$	A1	Both	
	$\frac{d^2V}{d\theta^2} = 4mga\cos\theta(2\cos\theta) - 4mga\sin\theta(2\sin\theta - 1)$	M1	Second derivative (or alternative method)	
	$V''\left(\frac{1}{2}\pi\right)(= -4mga) < 0 \Rightarrow$ unstable	M1	Consider sign	
		A1	One correct conclusion validly shown	
	$V''\left(\frac{1}{6}\pi\right)\left(= 4mga\cdot\frac{\sqrt{3}}{2}(\sqrt{3})\right) > 0 \Rightarrow$ stable	A1	Complete argument	7

3(i)	Mass of 'ring' $\approx 2\pi r\delta r\rho$	B1	May be implied	
	$\Rightarrow I_C = \int_0^a r^2 \cdot 2\pi\rho r \, dr$	M1	Set up integral	
		A1	All correct	
	$= \left[2\pi\rho \cdot \frac{1}{4}r^4\right]_0^a = \frac{1}{2}\pi a^4\rho$	M1	Integrate	
	$M = \pi a^2\rho$	M1	Use relationship between ρ and M	
	$\Rightarrow I_C = \frac{1}{2}Ma^2$	E1	Complete argument	
				6
(ii)	$I_A = I_C + M\left(\frac{1}{10}a\right)^2$	M1	Use parallel axis theorem	
	$= \frac{1}{2}Ma^2 + \frac{1}{100}Ma^2 = 0.51Ma^2$	E1	Convincingly shown	
				2
(iii)	$I_A\ddot{\theta} = -Mg \cdot \frac{1}{10}a\sin\theta$	B1	LHS	
		B1	RHS	
	$\Rightarrow \ddot{\theta} = -\frac{g}{5.1a}\sin\theta$	M1	Expression for $\ddot{\theta}$	
	θ small $\Rightarrow \sin\theta \approx \theta$	M1	Use small angle approximation	
	$\Rightarrow \ddot{\theta} = -\frac{g}{5.1a}\theta$, i.e. SHM	E1	Complete argument and conclude SHM	
	Period $2\pi\sqrt{\frac{5.1a}{g}} \approx 4.53\sqrt{a}$	F1	Follow their SHM equation	
				6
(iv)	e.g. 	B1	Show PAC in straight line (in any direction)	
	$mg \cdot \frac{9}{10}a = Mg \cdot \frac{1}{10}a$	M1	Moments or $(\Sigma m)\bar{x} = \Sigma mx$ (oe)	
	$\Rightarrow m = \frac{1}{9}M$	A1	Method may be implied	
	$I = 0.51Ma^2 + m\left(\frac{9}{10}a\right)^2$	M1		
	$= 0.6Ma^2$	E1	Convincingly shown	
				5
(v)	KE $= \frac{1}{2}I\omega^2 = \frac{1}{2}(0.6Ma^2)\omega^2$	M1	Attempt to find KE	
	$= 0.3Ma^2\omega^2$	A1		
	$C \cdot n \cdot 2\pi = 0.3Ma^2\omega^2$	M1	Work-energy equation	
		A1	Correct equation	
	$\Rightarrow C = \frac{0.3Ma^2\omega^2}{2n\pi}$	A1		
				5

4(i)	At terminal velocity, $\Sigma F = 0 \Rightarrow k \cdot 60^2 = 90g$ $\Rightarrow k = \frac{1}{40}g$	M1 E1	Equilibrium of forces Convincingly shown	
				2
(ii)	$90v\frac{dv}{dx} = 90g - \frac{1}{40}gv^2$	M1 A1	N2L	
	$\int \frac{90v}{90g - \frac{1}{40}gv^2} dv = \int dx$	M1	Separate and integrate	
	$-\frac{1800}{g}\ln\left\|90g - \frac{1}{40}gv^2\right\| = x + c_1$	A1	LHS	
	$90g - \frac{1}{40}gv^2 = Ae^{-\frac{gx}{1800}}$			
	$v^2 = \frac{40}{g}\left(90g - Ae^{-\frac{gx}{1800}}\right)$	M1	Rearrange, dealing properly with constant	
	$x = 0, v = 0 \Rightarrow A = 90g$	M1	Use condition	
	$v^2 = 3600\left(1 - e^{-\frac{gx}{1800}}\right)$	E1	Complete argument	
				7
(iii)	WD against $R = \int_0^{1800} kv^2 dx$	B1		
	$= \int_0^{1800} 90g\left(1 - e^{-\frac{gx}{1800}}\right) dx$			
	$= \left[90g\left(x + \frac{1800}{g}e^{-\frac{gx}{1800}}\right)\right]_0^{1800}$	M1	Integrate	
	$= 162000(g + e^{-g} - 1)$	A1		
	$x = 1800 \Rightarrow v^2 = 3600(1 - e^{-g})$	B1		
	Loss in energy $= 90g \cdot 1800 - \frac{1}{2} \cdot 90 \cdot 3600(1 - e^{-g})$	M1 M1	GPE KE	
	$= 162000(g + e^{-g} - 1) =$ WD against R	E1	Convincingly shown (including signs)	
				7
(iv)	$v = 60\sqrt{1 - e^{-g}} \approx 59.9983$	B1		
				1
(v)	$90\frac{dv}{dt} = 90g - 90v$	M1 A1	N2L	
	$\int \frac{dv}{g - v} = \int dt \quad \left[\text{or } \int_{59.9983}^{10} \frac{dv}{g - v} = \int_0^t dt\right]$	M1	Separate and integrate	
	$-\ln\|g - v\| = t + c_2$	A1		
	$t = 0, v = 59.9983 \Rightarrow c_2 = -3.91598$	M1	Use condition (or limits)	
	$v = 10 \Rightarrow t = -\ln 0.2 + 3.91598$	M1	Calculate t	
	≈ 5.53 s	A1		
				7

1(i)	$(m-\lvert\delta m\rvert)(v+\delta v)+\lvert\delta m\rvert(v-u)-mv=-mg\delta t$	M1	Impulse = change in momentum	
		A1	Accept sign errors in δm	
	$m\delta v-u\lvert\delta m\rvert-\lvert\delta m\rvert\delta v=-mg\delta t$			
	$m\frac{\delta v}{\delta t}+u\frac{\delta m}{\delta t}+\delta m\frac{\delta v}{\delta t}=-mg$	M1	Form DE	
	$\Rightarrow m\frac{dv}{dt}+u\frac{dm}{dt}=-mg$	E1	Complete argument (including signs)	
				4
(ii)	$\frac{dm}{dt}=-k\Rightarrow m=m_0-kt$	M1		
	So $(m_0-kt)\frac{dv}{dt}-uk=-(m_0-kt)g$	A1		
	$\frac{dv}{dt}=\frac{uk}{m_0-kt}-g$			
	$v=\int\left(\frac{uk}{m_0-kt}-g\right)dt$	M1	Integrate	
	$=-u\ln(m_0-kt)-gt+c$	A1		
	$t=0, v=0\Rightarrow 0=-u\ln m_0+c$	M1	Use condition	
	$v=-u\ln\left(1-\frac{k}{m_0}t\right)-gt$	A1		
	Fuel burnt when $m_0-kt=0.25m_0$	M1		
	$v=-u\ln 0.25-\frac{0.75m_0g}{k}$	A1		
				8

2(i)	$m\frac{dv}{dt} = -mkv^{\frac{3}{2}}$	M1	N2L	
		A1		
	$\int -v^{-\frac{3}{2}} dv = \int k\, dt$	M1	Separate and integrate	
	$2v^{-\frac{1}{2}} = kt + c$	A1		
	$t = 0, v = 25 \Rightarrow c = \frac{2}{5}$	M1	Use condition	
	$2v^{-\frac{1}{2}} = kt + \frac{2}{5}$	M1	Rearrange	
	$v = 4\left(kt + \frac{2}{5}\right)^{-2}$	E1		
				7
(ii)	$x = \int 4\left(kt + \frac{2}{5}\right)^{-2} dt$			
	$= -\frac{4}{k}\left(kt + \frac{2}{5}\right)^{-1} + A$	M1	Integrate	
	$t = 0, x = 0 \Rightarrow A = \frac{10}{k}$	M1	Use condition	
	$x = \frac{1}{k}\left(10 - \frac{4}{kt + \frac{2}{5}}\right)$	A1		
				3
(iii)	The speed decreases, tending to zero	B1		
	The displacement tends to $\frac{10}{k}$	B1	Cv $(10/k)$	
				2

3(i)	$V = -mga\sin\theta + \frac{\lambda}{2(2a)}(3a\sin\theta)^2$	M1	GPE term
		M1	EPE term
		A1	
	$\frac{dV}{d\theta} = -mga\cos\theta + \frac{\lambda}{4a}\cdot 9a^2 \cdot 2\sin\theta\cdot\cos\theta$	M1	Differentiate
		A1	
	$= a\cos\theta\left(\frac{9}{2}\lambda\sin\theta - mg\right)$	E1	
			6
(ii)	$\frac{dV}{d\theta} = 0 \Leftrightarrow \cos\theta = 0$ or $\sin\theta = \frac{2mg}{9\lambda}$	M1	Solve $\frac{dV}{d\theta} = 0$
(A)	$\lambda > \frac{2}{9}mg$		
	$\theta = \frac{\pi}{2}$	A1	
	and $\theta = \sin^{-1}\frac{2mg}{9\lambda}$	A1	
	$\frac{d^2V}{d\theta^2} = -a\sin\theta\left(\frac{9}{2}\lambda\sin\theta - mg\right) + a\cos\theta\left(\frac{9}{2}\lambda\cos\theta\right)$	M1	Second derivative (or other valid method)
		A1	Any correct form
	$= a\left(\frac{9}{2}\lambda(1-2\sin^2\theta) + mg\sin\theta\right)$		
	$V''\left(\frac{\pi}{2}\right) = a\left(-\frac{9}{2}\lambda + mg\right) < 0$	M1	Substitute $\theta = \frac{\pi}{2}$
	$\Rightarrow$ unstable	A1	Deduce unstable
	$V''\left(\sin^{-1}\left(\frac{2mg}{9\lambda}\right)\right) = a\left(\frac{9}{2}\lambda\left(1 - 2\left(\frac{2mg}{9\lambda}\right)^2\right) + \frac{2(mg)^2}{9\lambda}\right)$	M1	Substitute other value

	$= \frac{9}{2}\lambda a\left(1 - \left(\frac{2mg}{9\lambda}\right)^2\right)$			
	$\lambda > \frac{2}{9}mg \Rightarrow \left(\frac{2mg}{9\lambda}\right)^2 < 1 \Rightarrow V'' > 0$	M1	Consider second derivative	
	$\Rightarrow$ stable	A1	Complete argument	
				10
(B)	$\lambda < \frac{2}{9}mg \Rightarrow$	M1	Consider solutions	
	$\theta = \frac{\pi}{2}$ only	A1		
	$V''\left(\frac{\pi}{2}\right) = a\left(-\frac{9}{2}\lambda + mg\right) > 0$	M1	Consider second derivative	
	$\Rightarrow$ stable	A1	Complete argument	
				4
(C)	$\lambda = \frac{2}{9}mg$ gives $\theta = \frac{1}{2}\pi$ only (from both factors)	M1	Consider solutions	
		A1		
	$V''\left(\frac{\pi}{2}\right) = 0$			
	$V'\left(\frac{\pi}{2} - \epsilon\right) = (+)(-) = (-)$			
	$V'\left(\frac{\pi}{2} + \epsilon\right) = (-)(-) = (+)$	M1	Valid method	
	Hence stable	A1	Complete argument	
				4

4(i)	Mass of slice $\approx \rho\pi y^2 \delta x$	M1		
	So $I_{\text{slice}} \approx \frac{1}{2}(\rho\pi y^2 \delta x)y^2$	M1		
	$= \frac{1}{32}\rho\pi x^4 \delta x$	A1		
	So $I_{\text{cone}} \approx \int_0^{2a} \frac{1}{32}\rho\pi x^4 \mathrm{d}x$	M1		
	$= \left[\frac{1}{160}\rho\pi x^5\right]_0^{2a}$	A1	ft	
	$= \frac{1}{5}\pi\rho a^5$	A1		
	$\rho = \dfrac{M}{\frac{2}{3}\pi a^3}$	M1		
	$\Rightarrow I_{\text{cone}} = \frac{3}{10}Ma^2$	E1		
				8
(ii)	Mass of small cone $= \left(\frac{1}{2}\right)^3 M = \frac{1}{8}M$			
	Mass of frustum $= \frac{7}{8}M$	B1		
	$I_{\text{large cone}} = I_{\text{small cone}} + I$	M1		
	$\frac{3}{10}Ma^2 = \frac{3}{10}\left(\frac{1}{8}M\right)\left(\frac{1}{2}a\right)^2 + I$	M1	Moment of inertia of small cone	
	$\Rightarrow I = \frac{93}{320}Ma^2$			
	$\frac{7}{8}M = 2.8, a = 0.1 \Rightarrow I = 0.0093$	E1		
				4

(iii)	$C = I\ddot{\theta} \Rightarrow \ddot{\theta} = \frac{0.05}{0.0093}$	M1		
		A1		
	$t = \frac{10}{\ddot{\theta}} = 1.86$	M1		
		A1		
				4
(iv)	Centre of mass:			
	$\frac{7}{8}M\bar{x} + \frac{1}{8}M \cdot \frac{3a}{4} = M \cdot \frac{3a}{2}$	M1		
		A1		
	OG $= \bar{x} = \frac{45a}{28} = \frac{4.5}{28} \approx 0.1607$	A1	Any distance which locates G	
	i.e. G is $\frac{1.7}{28} \approx 0.0607$ m from the small circular face			
				3
(v)	$0.1J = I(10 - 5)$	M1	Moment of impulse = ang. momentum	
	$J = 0.465$	A1		
	Radius at G is $\frac{1}{2}\bar{x}$	B1		
	$\left(\frac{4.5}{56}\right)J = I(5 - \omega)$	M1	Moment of impulse = ang. momentum	
	$\Rightarrow \omega = \frac{55}{56} \approx 0.98$	A1		
				5

1(i)	$\frac{dm}{dt} = -\lambda m \Rightarrow m = m_0 e^{-\lambda t}$	M1		
		A1		
				2
(ii)	$\frac{d}{dt}(mv) = mg - kmv$	B1	N2L	
	$\frac{dm}{dt}v + m\frac{dv}{dt} = mg - kmv$	M1	Expand derivative	
	$-\lambda mv + m\frac{dv}{dt} = mg - kmv$	M1	Substitute	
	$\frac{dv}{dt} = g + (\lambda - k)v$	A1		
	$\int \frac{dv}{g+(\lambda-k)v} = \int dt$	M1	Separate and integrate	
	$\frac{1}{\lambda-k}\ln(g + (\lambda - k)v) = t + c$	A1√		
	$g + (\lambda - k)v = Ae^{(\lambda-k)t}$			
	$v = 0, t = 0 \Rightarrow A = g$	M1	Use condition	
	$v = \frac{g}{\lambda-k}\left(e^{(\lambda-k)t} - 1\right)$ AG	E1	Convincingly shown	
				8
(iii)	$m = \frac{1}{2}m_0 \Rightarrow e^{-\lambda t} = \frac{1}{2}$	M1	Accept substituted into their expression in part (i)	
	$\Rightarrow t = \frac{1}{\lambda}\ln 2$			
	$v = \frac{g}{\lambda-k}\left(2^{\frac{\lambda-k}{\lambda}} - 1\right)$	A1	Any correct form	
				2

2(i)	$V = \frac{1}{2}k(2a - x - a)^2 + \frac{1}{2}k\left(\sqrt{a^2 + x^2} - a\right)^2$	M1	for $E = \frac{1}{2}kx^2$	
		A1		
		A1		
	$\frac{dV}{dx} = -k(a - x)$ $+k\left(\sqrt{a^2 + x^2} - a\right) \cdot 2x \cdot \frac{1}{2}(a^2 + x^2)^{-1/2}$	M1		
	$= -k(a - x) + kx\left(1 - \frac{a}{\sqrt{a^2+x^2}}\right)$			
	$= 2kx - ka - \frac{kax}{\sqrt{a^2+x^2}}$ AG	E1	Convincingly shown	
				5
(ii)	$\frac{d^2V}{dx^2} = 2k - \frac{ka\sqrt{a^2 + x^2} - kax \cdot x(a^2 + x^2)^{-1/2}}{a^2 + x^2}$	M1		
	$= 2k - \frac{ka^3}{(a^2+x^2)^{3/2}}$	A1		
	$(a^2 + x^2)^{3/2} > (a^2)^{3/2} = a^3$	M1		
	$\Rightarrow \frac{ka^3}{(a^2 + x^2)^{3/2}} < k \Rightarrow V''(x) > 2k - k > 0$	E1	Convincingly shown	
				4
(iii)	$x = \frac{1}{2}a \Rightarrow V' = ka - ka - \frac{ka \cdot \frac{1}{2}a}{\sqrt{a^2+\left(\frac{1}{2}a\right)^2}} < 0$	M1		
	$x = a \Rightarrow V' = 2ka - ka - \frac{ka^2}{\sqrt{a^2+a^2}} = ka - \frac{ka}{\sqrt{2}} > 0$			
	Hence (as V' continuous) $V' = 0$ between $\frac{1}{2}a$ and a.	E1	Convincingly shown	
	So equilibrium. Stable as $V'' > 0$.	B1		
				3

3(i)	$800v\frac{dv}{dx} = \frac{8v^4}{v} - 8v^2$	M1	N2L with P/v	
		A1		
	$\int \frac{100dv}{v^2-v} = \int dx$	M1	Separate	
	$\int 100\left(\frac{1}{v-1} - \frac{1}{v}\right)dx = \int dx$	M1	Partial fractions	
	$100(\ln(v-1) - \ln v) = x + c$	A1		
	$x = 0, v = 2 \Rightarrow c = -100\ln 2$	M1	Use condition	
	$100\ln\left(\frac{2(v-1)}{v}\right) = x$	A1	AEF, condone *m*	
	$v = 20 \Rightarrow x = 100\ln\left(2 \times \frac{19}{20}\right) = 100\ln 1.9$	E1		
	$\frac{2(v-1)}{v} = e^{0.01x}$			
	$2v - 2 = ve^{0.01x}$	M1	Rearrange	
	$v = \frac{2}{2-e^{0.01x}}$	A1	Cao without *m*	
				10
(ii)	$\frac{dx}{dt} = \frac{2}{2-e^{0.01x}}$	M1		
	$\int(2 - e^{0.01x})dx = \int 2dt$	M1	Separate and integrate	
	$2x - 100e^{0.01x} = 2t + c_2$	A1		
	$x = 0, t = 0 \Rightarrow c_2 = -100$	M1	Use condition	
	$2x - 100e^{0.01x} = 2t - 100$	A1	Any correct form	
	$x = 100\ln 1.9 \Rightarrow t \approx 19.2$ AG	E1		
				6
(iii)	$800\frac{dv}{dt} = -8v^2$	M1	N2L	
		A1		
	$\int 100v^{-2}dv = \int -1dt$	M1	Separate and integrate	
	$-100v^{-1} = -t + c_3$	A1		
	$t = 19.2, v = 20 \Rightarrow -5 = -19.2 + c_3$	M1	Use condition	
	$c_3 = 14.2$			
	$v = \frac{100}{t-14.2}$	M1	Rearrange	
		A1	CAO	
	$2 = \frac{100}{t-14.2} \Rightarrow t = 64.2$	B1	Accept $t = 45$ (time for this part of motion)	
				8

4(i)	$I_x = \frac{1}{2}my^2$	B1	
	$2I_{\text{diameter}} = I_x$	M1	Use perpendicular axes theorem
	$I_{\text{diameter}} = \frac{1}{4}my^2$	B1	
	$I = \frac{1}{4}my^2 + mx^2$	M1	Use parallel axes theorem
	$= m\left(\frac{1}{4}\left(\frac{1}{2}x\right)^2 + x^2\right)$	M1	Use $y = \frac{1}{2}x$
	$= \frac{17}{16}mx^2$ AG	E1	Complete argument
			6
(ii)	Mass of slice $\approx M\left(\frac{\pi y^2 \delta x}{\frac{1}{3}\pi a^2 \cdot 2a}\right)$	M1	
	$= \frac{3M}{2a^3}y^2\,\delta x$	B1	Deal correctly with mass/density
	$I_{\text{slice}} \approx \frac{17}{16}\left(\frac{3M}{2a^3}y^2\delta x\right)x^2$	M1	
		A1	
	$= \frac{51M}{128a^3}x^4\,\delta x$	M1	Substitute for y
	$I = \int_0^{2a}\frac{51M}{128a^3}x^4\,\mathrm{d}x$	M1	
	$= \frac{51M}{128a^3}\left[\frac{1}{5}x^5\right]_0^{2a}$	A1	
	$= \frac{51}{20}Ma^2$ AG	E1	Complete argument
			8
(iii)	$\frac{1}{2}I\dot{\theta}^2 - Mg\cdot\frac{3}{2}a\cos\theta = -Mg\cdot\frac{3}{2}a\cos\alpha$	M1	Energy equation
		B1	Position of centre of mass
		A1	KE term
		F1	GPE terms ft their CoM only
	$\dot{\theta}^2 = \frac{3Mga}{I}(\cos\theta - \cos\alpha)$		
	$= \frac{20g}{17a}(\cos\theta - \cos\alpha)$	E1	Complete argument
			5
(iv)	$2\dot{\theta}\ddot{\theta} = -\frac{20g}{17a}\sin\theta\,\dot{\theta}$	M1	Differentiate or use $I\ddot{\theta}$ = torque
	$\ddot{\theta} = -\frac{10g}{17a}\sin\theta$	A1	
	$\approx -\frac{10g}{17a}\theta$ for small θ	M1	Use $\sin\theta \approx \theta$
	Hence SHM	E1	
	Period $2\pi\sqrt{\frac{17a}{10g}}$	B1	
			5

Question			Answer	Marks	Guidance
1	(i)		$mv=(m+\delta m)(v+\delta v)+(-\delta m)(v-u)$ (note $\delta m<0$)	M1 A1	Attempt at momentum equation Condone wrong sign of δm
			$mv=mv+v\delta m+m\delta v+\delta m\delta v-v\delta m+u\delta m$		
			$m\dfrac{\delta v}{\delta t}+u\dfrac{\delta m}{\delta t}+\delta m\dfrac{\delta v}{\delta t}=0$	M1	Simplify and divide by δt
			$m\dfrac{dv}{dt}=-u\dfrac{dm}{dt}$		
			$\dfrac{dm}{dt}=-k$	B1	
			$m=m_0-kt$	B1	SOI
			$(m_0-kt)\dfrac{dv}{dt}=uk$	E1	All correct including sign of δm
				[6]	
1	(ii)		$\int dv=\int\dfrac{uk}{m_0-kt}dt$	M1	Separate and integrate
			$v=-u\ln(m_0-kt)+c$	A1	
			$t=0, v=v_0 \Rightarrow v_0=-u\ln m_0+c$	M1	Use condition
			$c=v_0+u\ln m_0$	A1	
			$v=v_0+u\ln\left(\dfrac{m_0}{m_0-kt}\right)$	A1	aef
				[5]	
2	(i)		Let equilibrium extension be e		
			$mv\dfrac{dv}{dx}=mg-k(e+x)$	M1 A1	N2L All terms correct
			At equilibrium, $mg=ke$	B1	oe
			So $mv\dfrac{dv}{dx}=-kx$	E1	With evidence of working
				[4]	

Question			Answer	Marks	Guidance
2	(ii)		$\int mv\,\mathrm{d}v = \int -kx\,\mathrm{d}x$	M1	Solutions from SHM acceptable
			$\frac{1}{2}mv^2 = -\frac{1}{2}kx^2 + c$	A1	
			$x = a, v = 0 \Rightarrow 0 = -\frac{1}{2}ka^2 + c$	M1	
			$\frac{1}{2}mv^2 = \frac{1}{2}k\left(a^2 - x^2\right)$	A1	oe
			$v = -\sqrt{\frac{k}{m}\left(a^2 - x^2\right)}$	E1	AG Complete argument including justification for square root.
			($v < 0$ when moving up)	**[5]**	
2	(iii)		$\int \frac{1}{\sqrt{a^2 - x^2}}\,\mathrm{d}x = \int -\sqrt{\frac{k}{m}}\,\mathrm{d}t$	M1*	Solutions from SHM *NOT* acceptable
			$\arcsin\left(\frac{x}{a}\right) = -\sqrt{\frac{k}{m}}\,t + c_2$	A1	
			$x = a, t = 0 \Rightarrow \frac{1}{2}\pi = c_2$	DM1	Accept $c_2 = \arcsin 1$
			$x = a\sin\left(\frac{1}{2}\pi - \sqrt{\frac{k}{m}}\,t\right) = a\cos\left(\sqrt{\frac{k}{m}}\,t\right)$	A1	Either form
				[4]	
3	(i)		$l = 2a\sin\theta$	M1	OE eg $a\sqrt{2 - 2\cos 2\theta}$
			$V = \frac{\lambda}{2a}(2a\sin\theta - a)^2$	A1	EPE OE eg $\frac{\lambda}{2a}(a\sqrt{2 - 2\cos 2\theta} - a)^2$
			$\ldots + mga\cos 2\theta$	M1	Both terms
				A1	GPE OE eg $mga\sin(\frac{1}{2}\pi - 2\theta)$
			$\frac{\mathrm{d}V}{\mathrm{d}\theta} = -2mga\sin 2\theta + \frac{\lambda}{a}\left(2a\sin\theta - a\right)\cdot 2a\cos\theta$	M1	Differentiate
			$= -4mga\sin\theta\cos\theta + 2\lambda a\cos\theta\left(2\sin\theta - 1\right)$	M1	Use trigonometric identities as necessary
			$= 2a\cos\theta(2\lambda\sin\theta - 2mg\sin\theta - \lambda)$	E1	
				[7]	

Question			Answer	Marks	Guidance
3	(ii)		$\theta = \frac{1}{2}\pi \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}\theta} = 0 \times (\ldots) = 0$	M1	
			hence equilibrium	E1	
			$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -2a\sin\theta(2\lambda\sin\theta - 2mg\sin\theta - \lambda)$ $+2a\cos\theta(2\lambda\cos\theta - 2mg\cos\theta)$	M1 A1	Here or in (iii) or use sign method
			$\theta = \frac{1}{2}\pi \Rightarrow \frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -2a(2\lambda - 2mg - \lambda)$	M1	Use V'' or equivalent method
			So $\lambda < 2mg \Rightarrow \frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} > 0 \Rightarrow$ stable	E1	
			If $\cos\theta \neq 0$ $\frac{\mathrm{d}V}{\mathrm{d}\theta} = 0 \Leftrightarrow 2\lambda\sin\theta - 2mg\sin\theta - \lambda = 0$		
			$\Leftrightarrow \sin\theta = \frac{\lambda}{2\lambda - 2mg}$	M1	Consider other solutions
			But $\lambda < 2mg \Rightarrow 2\lambda - 2mg < \lambda$		
			$\Rightarrow \sin\theta > 1$ or $\sin\theta < 0$	M1	Attempt at showing not valid
			So no valid solutions	E1 **[9]**	Must consider both ends
3	(iii)		If $\lambda > 2mg, \theta = \frac{1}{2}\pi$ as before	B1	
			$V'' < 0$ so unstable	B1	
			or $\sin\theta = \frac{\lambda}{2\lambda - 2mg}$	E1	
			and $\frac{1}{2} < \frac{\lambda}{2\lambda - 2mg} < 1$ so gives valid solution	E1	
			$\theta = \sin^{-1}\left(\frac{\lambda}{2\lambda - 2mg}\right)$ or $\pi - \sin^{-1}\left(\frac{\lambda}{2\lambda - 2mg}\right)$	B1	For both
			and $V'' = 0 + 2a\cos^2\theta(2\lambda - 2mg)$	M1	
			$= (+\text{ve})(+\text{ve})$ so stable (in both cases)	A1 **[7]**	

Question			Answer	Marks	Guidance	
4	(i)		$\delta I = 2\pi r \delta r \rho r^2$	B1		
			$\rho = \dfrac{m}{\pi a^2}$	B1		
			$I_{\text{disc}} = \int_0^a \dfrac{m}{2a^2} r^3 \,\mathrm{d}r$	M1	for $k\int r^3 dr$	
			$= \dfrac{m}{2a^2}\left[\dfrac{1}{4}r^4\right]_0^a$	M1	$k\left[\frac{1}{4}r^4\right]_0^a$ with limits	
				A1	$\frac{k}{4}a^4$	
			$= \dfrac{1}{2}ma^2$	E1		
				[6]		
4	(ii)		$I = m_1 a^2$	M1	Curved surface	$2\pi h\rho a^3$
			$+ \frac{1}{2}ma^2 \times 2$	M1	Combine	$+ \frac{1}{2}\rho\pi a^4 \times 2$
			$m = M\dfrac{\pi a^2}{2\pi a^2 + 2\pi ah}$	B1		$m = \pi\rho a^2$
			$m_1 = M\dfrac{2\pi ah}{2\pi a^2 + 2\pi ah}$	B1		$m_1 = 2\pi ah\rho$
			So $I = Ma^2\left(\dfrac{\pi a^2 + 2\pi ah}{2\pi a^2 + 2\pi ah}\right)$	M1	Substitute	$I = M\dfrac{\pi\rho a^4 + 2\pi\rho a^3}{2\pi\rho a^2 + 2\pi\rho ah}$
			$I = \dfrac{1}{2}Ma^2\left(\dfrac{a+2h}{a+h}\right)$	E1		
				[6]		
4	(iii)		$I = \dfrac{1}{2}\times 8 \times 0.5^2\left(\dfrac{0.5+0.6}{0.5+0.3}\right) = 1.375$	B1		
			$I(\omega - 0) = Ja$ $1.375\omega = 55 \times 0.5$	M1	Impulse/momentum	
			$\omega = 20 \text{ rad s}^{-1}$	A1		
				[3]		

Question			Answer	Marks	Guidance
4	(iv)		$I\frac{\mathrm{d}\dot{\theta}}{\mathrm{d}t}=-2\dot{\theta}^2$	B1	
			$\int 1.375\dot{\theta}^{-2}\,\mathrm{d}\dot{\theta}=\int -2\,\mathrm{d}t$	M1	Separate
			$-\frac{1.375}{\dot{\theta}}=-2t+c$	M1 A1	Integrate
			$t=0,\dot{\theta}=20\Rightarrow c=-0.06875$	M1	Use condition
			$t=5\Rightarrow -\frac{1.375}{\dot{\theta}}=-10-0.06875$	M1	
			$\Rightarrow \dot{\theta}=0.137$ (3sf)	A1 **[7]**	
4	(v)		$I\left(\frac{-0.137}{t}\right)=-0.03$	M1	Complete method
				A1	with correct acceleration (or both sides +ve)
			$t=6.26$ s	A1 **[3]**	awfw [6.25, 6.3] CAO

Question			Answer	Marks	Guidance	
1	(i)		$\frac{dm}{dt} = k \Rightarrow m = kt + c$	B1		
			conditions $\Rightarrow m = m_0 + kt$	B1		
			$\left(\frac{d}{dt}(mv) = 0 \Rightarrow\right) \quad mv = m_0 v_0$	M1 A1	Momentum equation	Or derive a differential equation in only two variables
			$v = \frac{m_0 v_0}{m} = \frac{m_0 v_0}{m_0 + kt}$	A1		
			$x = \int \frac{m_0 v_0}{m_0 + kt} dt$	M1	Integrate their expression for v	
			$= \frac{m_0 v_0}{k} \ln(m_0 + kt) \quad (+ c_2)$	A1		
			conditions $\Rightarrow 0 = \frac{m_0 v_0}{k} \ln m_0 + c_2$	M1	Use initial conditions	
			so $x = \frac{m_0 v_0}{k}(\ln(m_0 + kt) - \ln m_0)$			
			$= \frac{m_0 v_0}{k} \ln\left(\frac{m_0 + kt}{m_0}\right) = \frac{m_0 v_0}{k} \ln\left(1 + \frac{kt}{m_0}\right)$	E1		
				[9]		
1	(ii)		$kt = 2m_0 \Rightarrow t = \frac{2m_0}{k} \Rightarrow v = \frac{1}{3} v_0$	B1	Follow through their $v = f(t)$	**SC** If $kt = m_0$ Award B1 either correct on follow through
			$x = \frac{m_0 v_0}{k} \ln 3$	B1	Ft	
				[2]		
2	(i)		$BC = 2 \times 0.5 \sin \frac{1}{2}\theta$	E1		
				[1]		

Question			Answer	Marks	Guidance	
2	(ii)		$V = -0.5g \cdot 0.25\sin\theta +$	M1	GPE	
			$\frac{1}{2}\cdot 2(\text{BC}-0.5)^2 + \frac{1}{2}\cdot 2(\text{BD}-0.5)^2$	M1	At least one EPE term	
			$\text{BD}^2 = 1^2 + 0.5^2 - 2\times 1\times 0.5\cos\theta = 1.25 - \cos\theta$	B1		
			$V = -1.225\sin\theta + (\sin\frac{1}{2}\theta - 0.5)^2 + (\sqrt{1.25-\cos\theta} - 0.5)^2$	A1	oe	
			$\frac{\mathrm{d}V}{\mathrm{d}\theta} = -1.225\cos\theta + 2(\sin\frac{1}{2}\theta - 0.5)(\frac{1}{2}\cos\frac{1}{2}\theta) +$	M1	Differentiate	
			$2\left(\sqrt{1.25-\cos\theta} - 0.5\right)\left(\frac{\sin\theta}{2\sqrt{1.25-\cos\theta}}\right)$	M1 A1	Use of chain rule one EPE term correct	
			$= -1.225\cos\theta + \sin\frac{1}{2}\theta\cos\frac{1}{2}\theta - 0.5\cos\frac{1}{2}\theta +$			
			$\sin\theta - \frac{0.5\sin\theta}{\sqrt{1.25-\cos\theta}}$			
			$= 1.5\sin\theta - 1.225\cos\theta - \frac{0.5\sin\theta}{\sqrt{1.25-\cos\theta}} - 0.5\cos\frac{1}{2}\theta$	E1	Complete argument	
				[8]		
2	(iii)		$\theta \approx 1.2$ and 4.1	B1	Both	Allow B1M1A1 from 1.1 and/or 4.05
			Stable and unstable respectively	B1		
			at $\theta \approx 1.2$, $\frac{\mathrm{d}V}{\mathrm{d}\theta}$ increasing because the graph shows that $\mathrm{f}'(\theta)$ is positive	M1	Consider gradient, relating f to $\frac{\mathrm{d}V}{\mathrm{d}\theta}$	
			so V minimum hence stable	A1	Clear evidence from the graph	
			at $\theta \approx 4.1$ $\frac{\mathrm{d}V}{\mathrm{d}\theta}$ decreasing because the graph shows that $\mathrm{f}'(\theta)$ is negative, so max. so unstable			
				[4]		

Question			Answer	Marks	Guidance	
3	(i)		$2\frac{dv}{dt} = \frac{2v^3 + 4v}{v} - 6v$	M1 A1	N2L	
			$\frac{dv}{dt} = v^2 - 3v + 2 = (1-v)(2-v)$	E1		
			$\int \frac{1}{(1-v)(2-v)} dv = \int dt$	M1	Separate	
			$\int \left(\frac{1}{1-v} - \frac{1}{2-v}\right) dv = \int dt$	M1	Partial fractions	
			$-\ln\lvert 1-v\rvert + \ln\lvert 2-v\rvert = t + c$	A1 A1	LHS RHS	
			$t = 0, v = 0 \Rightarrow \ln 2 = c$	M1	Use condition	
			$t = \ln(2-v) - \ln(1-v) - \ln 2$	M1	Rearrange	
			$= \ln \frac{2-v}{2(1-v)}$	E1		
				[10]		
3	(ii)		$\Rightarrow \frac{2-v}{2(1-v)} = e^t \Rightarrow 2 - v = 2e^t - 2e^t v$	M1	Rearrange	
			$v = \frac{2(e^t - 1)}{2e^t - 1}$	A1		
				[2]		
3	(iii)		$v = 0.8 \Rightarrow P = 2 \times 0.8^3 + 4 \times 0.8 = 4.224$	E1		
			$t = \ln \frac{2-0.8}{2(1-0.8)} = \ln 3 \approx 1.10$	B1		
				[2]		

Question			Answer	Marks	Guidance	
3	(iv)		$2\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{4.224}{v} - 6v$	B1	N2L	
			$\int \frac{2}{\frac{4.224}{v} - 6v}\mathrm{d}v = \int \mathrm{d}t$	M1	Separate	
			$\int \frac{v}{2.112 - 3v^2}\,\mathrm{d}v = \int \mathrm{d}t$	M1		
			$-\frac{1}{6}\ln\left\|2.112 - 3v^2\right\| = t + c_2$	A1	LHS	
				A1	RHS	
			$2.112 - 3v^2 = A\mathrm{e}^{-6t}$			**Alternate**
			$t = \ln 3, v = 0.8 \Rightarrow 2.112 - 3\times 0.8^2 = A\mathrm{e}^{-6\ln 3}$	M1	Use condition to find constant	
			$A = 139.968$	A1		
				M1	Rearrange to make v the subject	$t \to \infty \Rightarrow 2.112 - 3v^2 \to 0$
			$v = \sqrt{0.704 - 46.656\mathrm{e}^{-6t}}$	A1	Correct	
			$t \to \infty \Rightarrow v \to \sqrt{0.704} \approx 0.839$	A1	Ft their expression for v	
				[10]		
4	(i)		$\rho = \frac{m}{\frac{1}{2}a^2 \cdot \frac{\pi}{3}}$	B1	Or let ρ = 1 without lose of generality	
			element with radius x and ‘width’ δx :			
			$\delta m = \rho x \frac{\pi}{3}\delta x \Rightarrow \delta I = \rho x \frac{\pi}{3}\delta x \cdot x^2$	M1		
			$= \frac{2m}{a^2}x^3\delta x$	A1	Ft their ρ	
			$I = \int_0^a \frac{2m}{a^2}x^3\,\mathrm{d}x$	M1	Integrate	
			$= \frac{2m}{a^2}\left[\frac{x^4}{4}\right]_0^a$	A1		
			$= \frac{2m}{a^2}\left(\frac{a^4}{4}\right) = \frac{1}{2}ma^2$	E1		
				[6]		

Question			Answer	Marks	Guidance	
4	(ii)		$\dfrac{2a}{\pi}$	B1 **[1]**		
4	(iii)		$\frac{1}{2}I\dot{\theta}^2 - mg\left(\frac{2a}{\pi}\right)\cos\theta = -mg\left(\frac{2a}{\pi}\right)\cos\frac{2}{3}\pi$ OE $\frac{1}{2}ma^2\dot{\theta}^2 = 2mg\left(\frac{2a}{\pi}\right)\left(\cos\theta + \frac{1}{2}\right)$ $\dot{\theta}^2 = \dfrac{4g}{\pi a}(2\cos\theta + 1)$	M1 A1 A1 E1 **[4]**	Energy Two correct terms All correct	RHS: or $mg\left(\frac{2a}{\pi}\right)\cos\frac{1}{3}\pi$
4	(iv)		Max $\dot{\theta}$ when $\cos\theta = 1$ $\Rightarrow \dot{\theta}^2 = \frac{12g}{\pi a}$ Speed max. furthest from axis, so max speed $= a\sqrt{\frac{12g}{\pi a}} = \sqrt{\frac{12ag}{\pi}}$	M1 A1 M1 A1 **[4]**	 oe	
4	(v)		$2\ddot{\theta}\dot{\theta} = \dfrac{4g}{\pi a}\left(-2\sin\theta\,\dot{\theta}\right)$ $\ddot{\theta} = -\dfrac{4g}{\pi a}\sin\theta$	M1 A1 **[2]**	Differentiate with respect to time Or use $C = I\ddot{\theta}$	May be seen in (iv) May be seen in (iv)
4	(vi)		$J \cdot x = \pm I \cdot \frac{1}{2}\omega \pm I \cdot \omega$ $J \cdot \frac{3}{4}a = \frac{1}{2}ma^2\left(\frac{1}{2}\omega\right) - \frac{1}{2}ma^2(-\omega)$ $\theta = (-)\frac{1}{3}\pi \Rightarrow \omega^2 = \dfrac{4g}{\pi a}\left(2\left(\frac{1}{2}\right)+1\right) \Rightarrow \omega = \sqrt{\dfrac{8g}{\pi a}}$ $J = m\sqrt{\dfrac{8ag}{\pi}}$	M1 A1 B1 A1 **[4]**		

Question			Answer	Marks	Guidance	
4	(vii)		$\frac{1}{2}I\left(\frac{1}{2}\sqrt{\frac{8g}{\pi a}}\right)^2 - mg\left(\frac{2a}{\pi}\right)\cos\frac{1}{3}\pi = -mg\left(\frac{2a}{\pi}\right)\cos\theta$	M1 A1		
			$\Rightarrow \theta = \cos^{-1}\frac{1}{4} \approx 1.32$	A1 **[3]**	CAO	